MAT1322
Lecture 18

Partial derivatives and Higher derivatives

22/03/2017

Recall: $f(x,y) \to 2$ variables

$$\frac{\partial f}{\partial x} = f_x = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$$ 'partial derivative of f w.r.t. x at (a,b)'

$$\frac{\partial f}{\partial y} = f_y = \lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}$$ 'partial derivative of f w.r.t. y at (a,b)'

Ex 1 $f(x,y) = x^3y^2 + 4xy^3$: compute f_x, f_y

$f_x = (x^3)'y^2 + 4(x)'y^3 = \underline{3x^2 \cdot y^2 + 4y^3}$

$f_y = x^3(y^2)' + 4x(y^3)' = \underline{x^3 \cdot 2y + 4x \cdot 3y^2}$

Ex 2 $f(x,y) = \dfrac{3x+7y}{4x-2y}$

DOMAIN all $(x,y) \in \mathbb{R}\times\mathbb{R} = \mathbb{R}^2$ except:

$4x - 2y = 0$

$\frac{4}{2}x = y$

$\boxed{2x = y}$

$$\boxed{f_x =} \frac{(3x+7y)'(4x-2y) - (3x+7y)(4x-2y)'}{(4x-2y)^2}$$

$$= \frac{(3+0)(4x-2y) - (3x+7y)(4-0)}{(4x-2y)^2}$$

$$= \frac{3(4x-2y) - 4(3x+7y)}{(4x-2y)^2}$$

$$= \frac{12x - 6y - 12x - 28y}{(4x-2y)^2}$$

$$= \boxed{\frac{-34y}{(4x-2y)^2}}$$

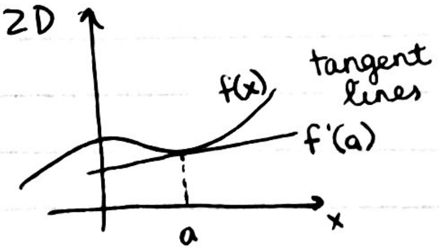
$$\boxed{f_y =} \frac{7(4x-2y) - (3x+7y)(-2)}{(4x-2y)^2}$$

$$= \frac{28x - 14y - (-6x - 14y)}{(4x-2y)^2}$$

$$\boxed{= \frac{34x}{(4x-2y)^2}}$$

Geometric interpretation

recall: 2D



$f'(a)$ is the slope of the tangent line at $x=a$.

now 3D * google a pic

f_x is the derivative along the x-direction → slope parallel to the x-axis. If we fix y and look at the cross-section curve, we can tell a lot about our function.

Higher Derivatives

$f_x \quad , \quad f_y$ ← these are functions in x & y : $f_x(x,y)$, $f_y(x,y)$

$f_{xx} \quad f_{xy} \qquad f_{yx} \quad f_{yy}$ we have 4 second derivatives!

<u>Ex</u> $f(x,y) = x^3 + x^2y^3 - 2y^2$ → all 2nd derivatives

need f_x and f_y first

$f_x = 3x^2 + 2x \cdot y^3 - 0 = \underline{3x^2 + 2xy^3}$

$f_y = 0 + 3y^2 \cdot x^2 - 4y = \underline{3x^2y^2 - 4y}$

$f_{xx} = \underline{6x + 2y^3}$

$f_{xy} = 0 + 3y^2 \cdot 2x = \underline{6xy^2}$

$f_{yy} = \frac{\partial}{\partial y}(3x^2y^2 - 4y) = 2y \cdot 3x^2 - 4 = \underline{6x^2y - 4}$

$f_{yx} = \frac{\partial}{\partial x}(3x^2y^2 - 4y) = 6x \cdot y^2 - 0 = \underline{6xy^2}$

$f_{xy} = f_{yx}$, isn't a coincidence!

Clairaut's Theorem

f defined on at least a disc D that contains (a,b).
If f_{xy} and f_{yx} are both continuous on D, then:

$$f_{xy}(a,b) = f_{yx}(a,b)$$